Model of Photovoltaic Power Plant with Constant Resistive Load

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***Abstract*—This paper deals with a mathematical model of a photovoltaic panel, which directly supplies a constant resistive load without inverter cooperation. Later on, this basic theory is used to develop a model of photovoltaic plant working with a constant resistive load. The model's correctness is evaluated through the comparison of simulated values and real measured data.**

***Keywords*—photovoltaic cell, solar panel, mathematic model, nonlinear equation, irradiation intensity.**

I. Introduction

Photovoltaic (PV) is the technology that deals with the direct conversation of irradiation energy into electrical energy. For the conversation are used large-scale semiconductor structures. In 1839 in Paris, Edmund Becquerel discovered the photovoltaic effect when he found out that some materials could be able to produce electricity if they are exposed by light. When the light stops, the electricity stops. The development of semiconductor technology in the fifties of the last century, P-N junction preparation and knowledge of physical processes in the PN junction created the conditions for the production of solar cells with a reasonable efficiency. For practical applications, a large number of photovoltaic cells are interconnected and encapsulated into units called photovoltaic panels. The advantage of photovoltaic panels is that they are working safety, quietly and they need no fuel, produce no waste, in most installations have no moving parts and therefore need a minimal maintenance [1], [2], [6].

II. Matematical Model of a PV Cell

The diode represents the P-N junction of solar cell. According to the polarity of the external DC voltage supply, the current flows through diode or does not flow. Size of the diode current is given by Shockley equation:

$$I_{\mathrm{D}} = I_{\mathrm{S}} \cdot \left[e^{\left(\frac{q \cdot U_{\mathrm{D}}}{k \cdot T}\right)} - 1 \right] \tag{1}$$

where I_{D} is a diode current, I_{S} is a diode saturation current, U_{D} is a diode voltage, q is an electron charge (1.6×10^{-19} C), k is a Boltzman constant (1.38×10^{-23} J/K) and T is a temperature dependence of the diode saturation current [3], [4].

The equivalent circuit of solar cell based on Shockley equation is shown in Fig. 1. This model consists from a current source, a diode, a shunt resistance R_{SH} and a series resistance R_{S}, which represents an internal resistance of the cell.

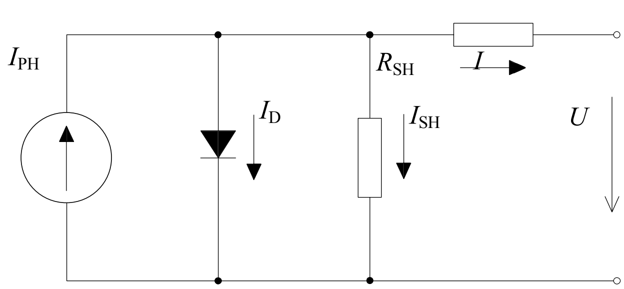

Fig. 1. Equivalent model of photovoltaic cell

Volt-Ampere characteristic is given by following equation

$$I = I_{\mathrm{PH}} - I_{\mathrm{S}} \cdot \left(e^{\left[\frac{q \cdot (U + I \cdot R_{\mathrm{S}})}{k \cdot T \cdot A}\right]} - 1 \right) - \frac{U + I \cdot R_{\mathrm{S}}}{R_{\mathrm{SH}}}, \tag{2}$$

where I_{PH} is a photo current (light generated current), U is the cell output voltage, A is an ideality factor. The photo current depends on solar irradiance and the temperature of solar cell which is given by [3]

$$I_{\mathrm{PH}} = \lambda \cdot \left[I_{\mathrm{SC}} + K_{\mathrm{I}} \cdot (T - T_{\mathrm{r}}) \right], \tag{3}$$

where I_{SC} is the short-circuit current of the photovoltaic cell at temperature 25°C and $\lambda = 1$ kW/m^2, K_{I} is a temperature coefficient of the cell, T_{r} is a reference temperature of the cell and λ is solar irradiance (kW/m^2). The saturation current of photovoltaic cell depends on temperature and it is given by [3]

$$I_{\mathrm{S}} = I_{\mathrm{RS}} \cdot \left(\frac{T}{T_r} \right)^3 \cdot e^{\left[q \cdot E_{\mathrm{G}} \cdot \frac{(1/T_{\mathrm{r}} - 1/T)}{K \cdot A} \right]}, \tag{4}$$

where I_{RS} is reverse saturation current depending on reference temperature and irradiation energy, E_{G} is a bandgap voltage for silicon. Ideality factor A depends on production technology of the photovoltaic cells [3]. A dependence of reverse saturation current on reference temperature is given by following equation

$$I_{\mathrm{RS}} = \frac{I_{\mathrm{SC}}}{e^{\left(\frac{q \cdot U_{\mathrm{OC}}}{k \cdot A \cdot T}\right)} - 1}, \tag{5}$$

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where U_{OC} is open-circuit voltage of the photovoltaic cell. The -1 term in (1) can be omitted, when the forward voltage is greater than about 0.1 volts at room temperature [5], [6].

In this paper, it is considered that the solar cell is loaded by known resistive load (Fig. 2). Current-voltage characteristic (I-V curve) of a solar cell is a nonlinear function, therefore the mathematical model is also nonlinear and it's necessary to use iterative methods (e.g. Newton's iterative method) to solve the model.

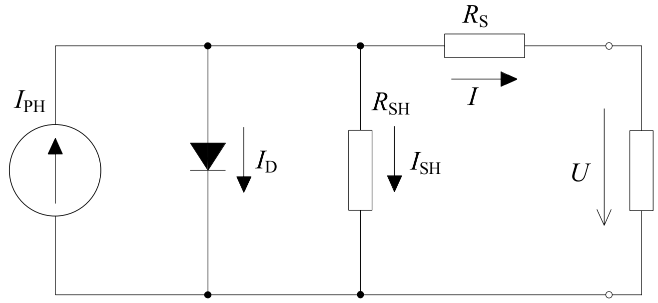

Fig. 2. Equivalent model of a photovoltaic cell with resistive load

Based on Fig. 2, the current supplied to the load is

$$I = I_{PH} - I_D - I_{SH} \ , \tag{6}$$

where I is an output current, which is possible to express by using a diode voltage U_D and a series combination of resistances R_S and R_Z:

$$\frac{U_D}{R_S + R_Z} + \frac{U_D}{R_{SH}} = I_{PH} - I_D \tag{7}$$

and so it can be written as

$$U_D = \left(I_{PH} - I_S \cdot e^{\frac{c \cdot U_D}{k \cdot T \cdot A}} \right) \cdot \frac{(R_s + R_z) \cdot R_{SH}}{R_S + R_Z + R_{SH}} . \tag{8}$$

We can simplify equation (8) using following substitutions:

$$K_E = \frac{q}{k \cdot T \cdot A} , \tag{9}$$

$$K_R = \frac{(R_S + R_Z) \cdot R_{SH}}{R_S + R_Z + R_{SH}} , \tag{10}$$

which simplify (8) to

$$f_{(U_D)} = U_D - \left(I_{PH} - I_s \cdot e^{K_E \cdot U_D} \right) \cdot K_R \tag{11}$$

the first derivation can be written

$$f'_{(U_D)} = 1 - I_S \cdot K_E \cdot K_R \cdot e^{K_E \cdot U_D} , \tag{12}$$

Then we can solve the nonlinear system (11) using Newton-Raphson iterative method:

$$U_D^{(k+1)} = U_D^{(k)} - \frac{f_{(U_D^{(k)})}}{f'_{(U_D^{(k)})}} . \tag{13}$$

As it can be seen on Fig. 3, the I-V curve has no local extremes. Therefore, gradient based methods, like N-R, are unable to find the solution of such an equation. This can be solved by modifying the original equation to a quadratic form. This alternated equation has the same solution as the original function, but now the solution is located in a local minimum of the function (Fig. 4).

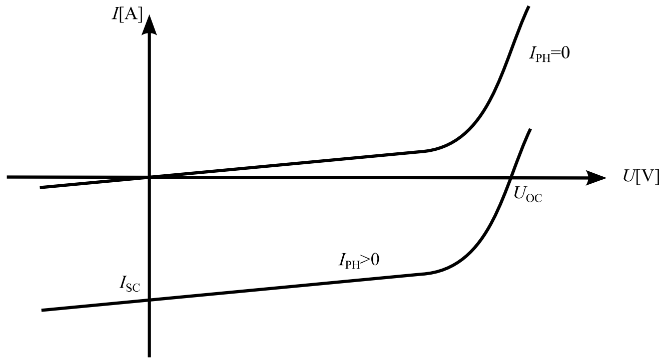

Fig. 3. The original I-V curve

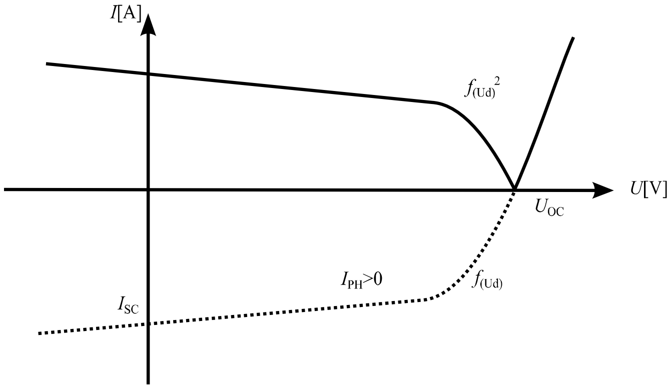

Fig. 4. The quadratic form of I-V curve

The quadratic form of (11) is

$$f_{(U_D)} = \left(U_D - \left(I_{PH} - I_s \cdot e^{K_E \cdot U_D} \right) \cdot K_R \right)^2 , \tag{14}$$

and its first derivation, used in (13), is

$$f'_{(U_D)} = 2 \cdot \left(K_R + I_S \cdot K_E \cdot e^{K_E \cdot U_D} + 1 \right) \cdot \\ \cdot \left(U_D - I_{PH} + K_R \cdot U_D + I_S \cdot e^{K_E \cdot U_D} \right) \tag{15}$$

Equation (15) changes all negative values to positive ones (Fig. 4) and so the function can be solved using the Newthon-Raphson iterative method (13). After knowing the solution – the value of diode voltage U_D, it is possible to calculate the values of all other variables. For example, if the resistance of the load R_z = 1 kΩ, the value of voltage on diode will be U_D = 0.595 V.

Model of photovoltaic module)

The power supplied by a single cell is very small. Electrical power is given by a total area of cell and its efficiency. To obtain a higher power, it is necessary to connect multiple cells to larger units called modules (panels). Cells building a panel can be connected both serial and parallel. A serial connection of cells increases only the output voltage as well as a parallel connection increases only the output current.

To simplify the circuit for the analysis of serial-parallel connections of multiple cells, the current sources in each cell were transformed to voltage sources (Fig. 5). Thus the voltage U_{PH} can be expressed as

$$U_{PH} = R_{SH} \cdot I_{PH} \,. \tag{16}$$

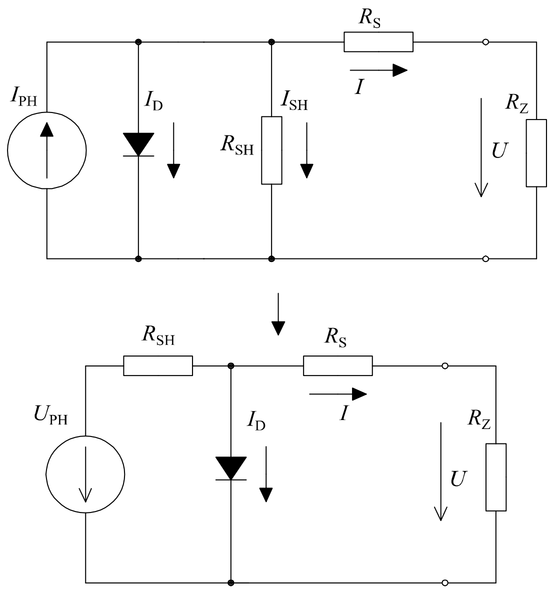

Fig. 5. The transformation of a current source to a voltage one

A serial combination of cells is illustrated in Fig. 6 and a parallel combination of cells is illustrated in Fig. 7.

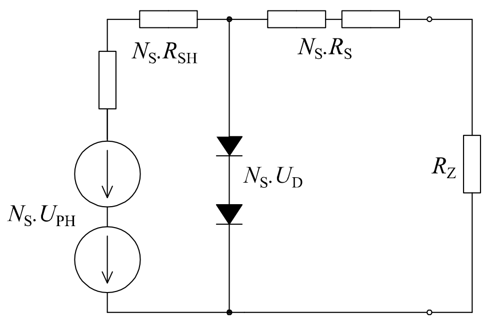

Fig. 6. Serial combination of cells

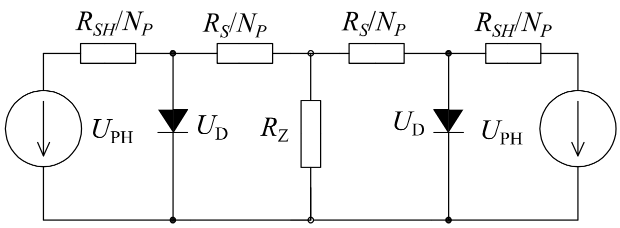

Fig. 7. Parallel combination of cells

The application of cells' serial combination on Fig. 6 to (16) results in

$$N_S \cdot U_{PH} = N_S \cdot R_{SH} \cdot \left(I_S \cdot e^{K_E \cdot U_D} + \frac{N_S \cdot U_D}{N_S \cdot R_S + R_Z} \right), \tag{17}$$

as well as the application of parallel combination on Fig. 7 results in

$$U_{PH} = \frac{R_{SH}}{N_P} \cdot \left(N_P \cdot I_S \cdot e^{K_E \cdot U_D} + \frac{U_D}{\frac{R_S}{N_P} + R_Z} \right). \tag{18}$$

The combination of (17) and (18) defines an equation for serial-parallel connection of cells:

$$N_S \cdot U_{PH} = \frac{N_S \cdot R_{SH}}{N_P} \cdot \left(N_P \cdot I_S \cdot e^{K_E \cdot U_D} + \frac{N_S \cdot U_D}{\frac{N_S \cdot R_S}{N_P} + R_Z} \right). \tag{19}$$

Equation (19) and its first derivation were used to create a simulation model in Matlab (Fig. 8).

```
while true
         ii=0;
         Ud=0.595;
         kk=kk+1;
         lam=n(kk,1)*0.001;
         T(kk,2)=Tr-25+T(kk,2);
         Iph=lam*(Isc+Ki*(T(kk,2)-Tr));
         Irs=Isc/(exp((q*Uoc)/(k*A*T(kk,2)))-1);
         Is=Irs*(T(kk,2)/Tr)^3*exp(q*Eg*((1/Tr-1/T(kk,2))/(k*A)));
         Ke=q/(k*T(kk,2)*A);
         Uph=Iph*Rsh;
         Kr=1/(Rz+(Rs*Ns/Np));
             while 2>1
               ii=ii+1;
               Udo=Ud;
              y1=(Ns*Uph-Ns*Rsh*Is*exp(Ke*Ud)- (Rsh/Np)*Ns*Kr*Ud*Ns)^2;
             dy1=2*((Kr*Ns^2*Rsh)/Np+Is*Ke*Ns*Rsh*exp(Ke*Ud))*(Is*Ns*Rs
             h*exp(Ke*Ud)-Ns*Uph+(Kr*Ns^2*Rsh*Ud)/Np);
                 Ud=Ud-y1/dy1;
                 if (abs(Ud-Udo)<eps)
                     break;
                  elseif (ii>5000)
                     Ud=NaN;
                 break;
                 end
              end
         if (kk==h)
             break;
         end
                 I=Ns*Ud/(Rs*Ns/Np+Rz);
                 U=Rz*I;
end
```

Fig. 8. Part of source code in Matlab

III. THE MEASUREMENT ON A REAL PV PANEL

To verify the results of introduced simulation model, we performed a measurement of the I-V curve of a small installation consisting of two series connected photovoltaic panels. One panel consists of two parallel branches of 60 series-connected cells.

TABLE I.
MEASURED DATA

I (A)	0	0.36	0.42	0.53	0.61	0.76	0.95	1.04
U (V)	70.5	69.6	69.5	69.3	69.1	68.8	68.5	68.3
I (A)	1.14	1.27	1.46	1.65	1.85	1.94	2.17	2.64
U (V)	68.1	67.9	67.5	67.2	66.8	66.7	66.2	65.2
I (A)	2.78	3.75	4.33	4.98	5.54	5.78	5.96	6.5
U (V)	64.9	62.7	61	58.1	54.2	33.2	22.9	0

During the measurement, the panel temperature was 23°C and solar irradiance was 0.83 kW/m^2. Data from measurement are summarized in Tab. I. Basic technical parameters of PV panels are shown in Tab. II.

TABLE II.
CATALOGE DATA AND CONSTANTS OF PHOTOVOLTAIC PANEL

I_{sc}	Short-circuit current	7.71 (A)
K_I	temperature coefficient of cell	0.11 (mA/C)
K	Boltzmann's constant	$1.38065 \cdot 10^{-23}$ (J/K)
T_r	termodynamic reference temperature	298.15 (K)
q	electron charge	$1.6 \cdot 10^{-19}$ (C)
E_g	bandgap voltage for silicon	1.11 (eV)
A	ideality factor	1.3 (-)
U_{oc}	Open-circuit voltage	0.589 (V)
R_s	Serial resistance	0.01136 (Ω)
R_{sh}	Parallel resistance	116.8415 (Ω)

IV. VERIFICATION OF SIMULATION MODEL

Based on equations (1) to (19), a simulation model in Matlab was created. The temperature and the solar irradiance were set according to conditions during the real measurement (λ = 0.830 kW/m^2 and T= 23 °C). Panel parameters were set according to catalog data provided by the manufacturer of the panels. These values are summarized in Tab. II.

The created simulation model was used to calculate I-V curve (Fig. 9) as well as a power curve (Fig. 10). Both figures show the comparison with measured data, as well.

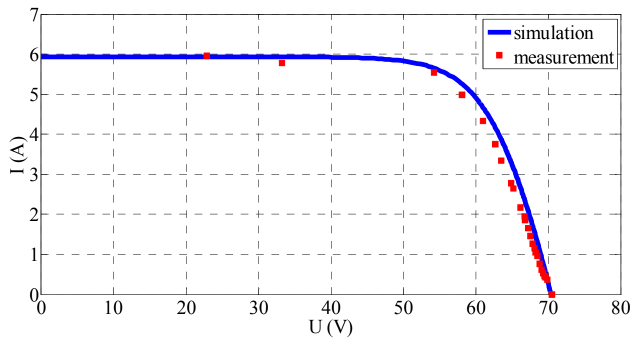

Fig. 9. Modelled and measured I-V curve

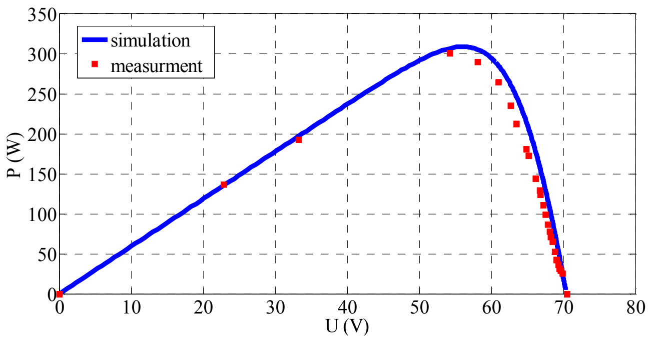

Fig. 10. Modelled and measured power curve

As it can be seen from Fig. 9 and Fig. 10, there is a very good correlation of simulated and measured data. Therefore, the verified simulation model was used to simulate the operation of measured photovoltaic power plant.

Modeling of PV panel operation

The installation supplies power to a constant resistive load with resistance R_z = 10 Ω. Measured data of solar irradiance and temperature, with 1-minute scale, were used as input values for the simulation model to calculate the power curve for a whole day operation (Fig. 11).

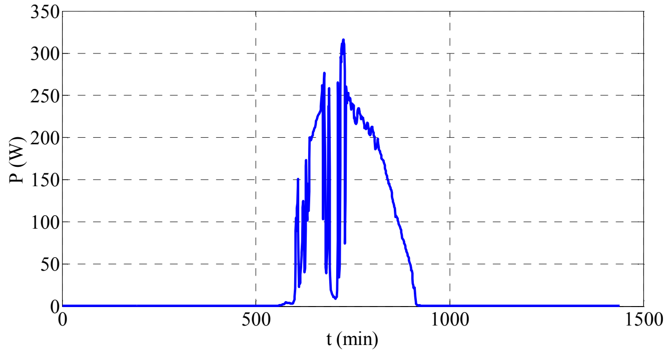

Fig. 11. Power output of PV panel simulation model

V. CONCLUSION

The paper describes a nonlinear mathematical model of PV panel consisting of series–parallel connected PV modules that was used to describe and then to model the operation of PV power plant feeding a constant resistive load. Model's accuracy was tested through the comparison of simulated and measured data, which were obtained by the measurement of a small installation consisting of two series connected photovoltaic panels. Modeled and simulated result showed a very good coincidence. Therefore, the created model could be used in future simulations of small local PV installations.

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